1. A particle P of mass m is above the surface of the Earth at distance x from the centre of the Earth. The Earth exerts a gravitational force on P. The magnitude of this force is inversely proportional to x^2.

At the surface of the Earth the acceleration due to gravity is g. The Earth is modelled as a sphere of radius R.

(a) Prove that the magnitude of the gravitational force on P is $\dfrac{mgR^2}{x^2}$.

(3)

A particle is fired vertically upwards from the surface of the Earth with initial speed $3U$. At a height R above the surface of the Earth the speed of the particle is U.

(b) Find U in terms of g and R.

(7)
(Total 10 marks)

2. A rocket is fired vertically upwards with speed U from a point on the Earth's surface. The rocket is modelled as a particle P of constant mass m, and the Earth as a fixed sphere of radius R. At a distance x from the centre of the Earth, the speed of P is v. The only force acting on P is directed towards the centre of the Earth and has magnitude $\dfrac{cm}{x^2}$, where c is a constant.

(a) Show that $v^2 = U^2 + 2c\left(\dfrac{1}{x} - \dfrac{1}{R}\right)$.

(5)

The kinetic energy of P at $x = 2R$ is half of its kinetic energy at $x = R$.

(b) Find c in terms of U and R.

(3)
(Total 8 marks)

3. A particle P of mass $\frac{1}{3}$ kg moves along the positive x-axis under the action of a single force. The force is directed towards the origin O and has magnitude $\frac{k}{(x+1)^2}$ N, where $OP = x$ metres and k is a constant. Initially P is moving away from O. At $x = 1$ the speed of P is 4 m s^{-1}, and at $x = 8$ the speed of P is $\sqrt{2}$ m s^{-1}.

(a) Find the value of k.

(10)

(b) Find the distance of P from O when P first comes to instantaneous rest.

(4)
(Total 14 marks)

4. A particle P of mass m kg slides from rest down a smooth plane inclined at 30° to the horizontal. When P has moved a distance x metres down the plane, the resistance to the motion of P from non-gravitational forces has magnitude mx^2 newtons.

Find

(a) the speed of P when $x = 2$,

(7)

(b) the distance P has moved when it comes to rest for the first time.

(3)
(Total 10 marks)

5. Above the earth's surface, the magnitude of the force on a particle due to the earth's gravity is inversely proportional to the square of the distance of the particle from the centre of the earth. Assuming that the earth is a sphere of radius R, and taking g as the acceleration due to gravity at the surface of the earth,

(a) prove that the magnitude of the gravitational force on a particle of mass m when it is a distance x ($x \geq R$) from the centre of the earth is $\frac{mgR^2}{x^2}$.

(3)

A particle is fired vertically upwards from the surface of the earth with initial speed u, where $u^2 = \frac{3}{2}gR$. Ignoring air resistance,

(b) find, in terms of g and R, the speed of the particle when it is at a height $2R$ above the surface of the earth.

(7)
(Total 10 marks)

6. A toy car of mass 0.2 kg is travelling in a straight line on a horizontal floor. The car is modelled as a particle. At time $t = 0$ the car passes through a fixed point O. After t seconds the speed of the car is v m s^{-1} and the car is at a point P with $OP = x$ metres. The resultant force on the car is modelled as $\frac{1}{10}x(4 - 3x)$ N in the direction OP. The car comes to instantaneous rest when $x = 6$. Find

(a) an expression for v^2 in terms of x,

(7)

(b) the initial speed of the car.

(2)
(Total 9 marks)

7. A car of mass 800 kg moves along a horizontal straight road. At time t seconds, the resultant force acting on the car has magnitude $\dfrac{48\,000}{(t+2)^2}$ newtons, acting in the direction of motion of the car. When $t = 0$, the car is at rest.

(a) Show that the speed of the car approaches a limiting value as t increases and find this value.

(6)

(b) Find the distance moved by the car in the first 6 s of its motion.

(6)
(Total 12 marks)

1. (a)

$$F = (-)\frac{k}{x^2}$$ M1

$$mg = (-)\frac{k}{R^2}$$ M1

$$F = \frac{mgR^2}{x^2} \quad *$$ A1 3

(b)

$$m\ddot{x} = -\frac{mgR^2}{x^2}$$ M1

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{gR^2}{x^2}$$ M1

$$\frac{1}{2}v^2 = \int\left(-\frac{gR^2}{x^2}\right)\mathrm{d}x$$ M1 dep on 1st M Mark

$$\frac{1}{2}v^2 = \frac{gR^2}{x} \quad (+c)$$ A1

$x = R,\ v = 3U$ $$\frac{9U^2}{2} = gR + c$$ M1 dep on 3rd M Mark

$$\frac{1}{2}v^2 = \frac{gR^2}{x} + \frac{9U^2}{2} - gR$$

$x = 2R,\ v = U$ $$\frac{1}{2}U^2 = \frac{gR^2}{2R} + \frac{9U^2}{2} - gR$$ M1 dep on 3rd M Mark

$$U^2 = \frac{gR}{8}$$

$$U = \sqrt{\frac{gR}{8}}$$ A1 7

[10]

2. (a) N2L $ma = -\dfrac{cm}{x^2}$ B1

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = -\frac{c}{x^2} \quad \Rightarrow \quad \frac{1}{2}v^2 = A + \frac{c}{m}$$ ignore A M1 A1

$$v^2 = B + \frac{2c}{m}$$

$x = R, v = U \quad \Rightarrow \quad B = U^2 - \dfrac{2c}{R}$ M1

Leading to $v^2 = U^2 + 2c\left(\dfrac{1}{x} - \dfrac{1}{R}\right)$ * cso A1 5

	(b)	$\frac{1}{2}\left[\frac{1}{2}mU^2\right] = \frac{1}{2}m\left[U^2 + 2c\left(\frac{1}{2R} - \frac{1}{R}\right)\right]$	M1 A1	
		Leading to $c = \frac{1}{2}RU^2$	A1	3

[8]

3.	(a)	$\frac{1}{3}\ddot{x} = -\frac{k}{(x+1)^2}$	M1	
		$\frac{1}{3}v\frac{dv}{dx} = -\frac{k}{(x+1)^2}$	M1	
		$\int v\,dv = \int -\frac{3k}{(x+1)^2}\,dx$		
		$\frac{1}{2}v^2 = \frac{3k}{x+1}$ $(+C)$	M1 A1=A1	
		Separating variables & attempting integration of both sides		
		$v^2 = \frac{6k}{x+1} + A$		
		Using boundary values to obtain two simultaneous equations.	M1	
		(1, 4) $16 = 3k + A$	A1	
		(8, √2) $2 = \frac{2k}{3} + A$	A1	
		$14 = \frac{7}{3}k \Rightarrow k = 6$	M1 A1	10
	(b)	$A = -2$	B1	
		$v^2 = \frac{36}{x+1} - 2 = 0$	M1	
		$x = 17$ (m)	M1 A1	4

[14]

4.	(a)	$mg \sin 30° - mx^2 = ma$	M1 A1	
		$\frac{g}{2} - x^2 = v\frac{dv}{dx}$ or $\frac{d\left(\frac{1}{2}v^2\right)}{dx}$	M1	
		$\frac{gx}{2} - \frac{x^3}{3}(+C) = \frac{v^2}{2}$	dep. M1 A1	
		$x = 2 : g - \frac{8}{3} = \frac{v^2}{2}$	dep. M1	
		$v = 3.8\text{ms}^{-1}$ (3.78)	A1	7
		Third M1 for attempting to integrate		

	(b)	$v = 0$: $\frac{gx}{2} - \frac{x^3}{3} = 0$	M1	
		$x^2 = \frac{3g}{2} \Rightarrow x = 3.8, (3.83), \sqrt{\frac{3g}{2}}$	dep. M1 A1 c.s.o	3
		Must have integrated for first M1		
				[10]

5.	(a)	$F = \frac{k}{x^2}$ [k may be seen as Gm_1m_2 , for example]	M1	
		Equating F to mg at $\boldsymbol{x = R}$, [mg = $\frac{k}{R^2}$]	M1	
		Convincing completion [$k = m$gR^2] to give $F = \frac{m\text{g}R^2}{x^2}$ *	A1	3
	(b)	Equation of motion: $(m)a = (–)\frac{(m)gR^2}{x^2}$; $(m)v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{(m)gR^2}{x^2}$	M1;M1	
		Integrating: ½$v^2 = \frac{gR^2}{x}$ (+ c) or equivalent	M1A1	
		Use of $v^2 = \frac{3gR}{2}$, $x = R$ to find c [c = – ¼gR] or use in def. int.	M1	
		Substituting $x = 3R$ and finding V; $\boldsymbol{V} = \sqrt{\frac{\mathbf{gR}}{\mathbf{6}}}$	M1;A1	7
				[10]
		Alternative in(b)		
		Work/energy $(–)\int_R^a \frac{mgR^2}{x^2}\mathrm{d}x$; = ½mv^2 – ½mu^2	M1;M1	
		Integrating: [$\frac{mgR^2}{x} - \frac{mgR^2}{R}$] = ½mv^2 – ½$m\frac{3\text{g}R}{2}$	M1A1M1	
		Final 2 marks as scheme	M1A1	

6.	(a)	$\frac{1}{10}x(4 - 3x) = 0.2\,a$	M1 A1	
		$\frac{1}{10}x(4 - 3x) = 0.2v\frac{\mathrm{d}v}{\mathrm{d}x}$ or $\frac{1}{10}x(4 - 3x) = 0.2\,\frac{\mathrm{d}(\frac{1}{2}v^2)}{\mathrm{d}x}$	M1	
		Integrating : $v^2 = 2x^2 - x^3$ (+ C) or equivalent	M1 A1	
		Substituting $x = 6$, $v = 0$ to find candidate's "C"	M1	
		$v^2 = 2x^2 - x^3 + 144$	A1	7
	(b)	Substituting $x = 0$ and finding v; $v = 12$ (m s^{-1})	M1; A1 ft	2
				[9]

7.	(a)	$800\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{48000}{(t+2)^2}$	M1	
		$v = 60 \displaystyle\int \dfrac{\mathrm{d}t}{(t+2)^2} = \dfrac{-60}{(t+2)}\ (+\ c)$	M1 A1	
		$t = 0,\ v = 0 \Rightarrow c = 30$	M1 A1	
		$v = 30 - \dfrac{60}{(t+2)} \Rightarrow v \to 30$ as $t \to \infty$	A1	6
	(b)	$s = \displaystyle\int v\,\mathrm{d}t = 30t - 60 \ln (t + 2)\ (+\ c)$	M1 A1	
		substitute in $t = 0$ and $t = 6$	M1	
		$s = 180 - 60 \ln 8,\ - -60 \ln 2$	A1, A1	
		≈ 96.8 m	A1	6

[12]

1. Part (a) should have posed no problems for the majority of candidates as it was a piece of standard bookwork. However, many candidates had little or no idea of how to proceed, but still managed to arrive at the printed result.

In part (b) many forgot the initial minus sign and ended up with a negative u^2. They did not seem to realise that the most likely error is a missing minus and instead, obtained a real value for the square root of their negative answer. Several tried to use energy with PE as *mgh* even though the result from part (a) should have told them that the force was variable and so this approach was invalid. There were a few "suvat" equations which again were invalid and a few successful energy attempts using integration. The majority of successful candidates adopted an indefinite integral approach; a few using definite integrals got the limits the wrong way round.

2. Part (a) proved difficult for very many candidates and there were a number of frequently repeated low scoring efforts. Instead of using integration, many tried to fit $Gm_1m_2/r^2 = F$ into the problem or quoted $a = c/x^2$ and then tried to use $v^2 = u^2 + 2$ as with some elaborate fiddling where $s = (R – x)$ cancelled with x^2 to give the correct answer. Another common error was to quote conservation of energy with PE rather than finding Work Done using integration. Part (b) was attempted more successfully but the positioning of the extra ½ in the KE equation was as often wrong as right. Candidates also had more trouble than they should have had in simplifying $\frac{1}{2R} - \frac{1}{R}$ accurately.

3. For many, this proved the easiest question on the paper and full marks were common. The most frequently seen error was the omission of the minus sign when setting up the initial differential equation. This produced the wrong value of *k* but did in fact give the correct relation between *v* and *x*, $v^2 = \frac{36}{x+1} - 2$, and this was followed through for full marks in part (b). The commonest manipulative error seen was the incorrect integration of $\frac{k}{(x+1)^2}$. A few candidates used definite rather than indefinite integration. These tended to be very well prepared candidates and they nearly always gained full marks.

4. Most candidates tackled this question confidently and successfully with evidence of being well drilled in the techniques involved. The most common mistakes seen were failure to recognise the acceleration as $v\,dv/dx$ and then integrating a with respect to x to get v, taking work as mx and an apparent failure to link the parts of the parts of the question – scoring well on one part only. A few candidates tried to use constant acceleration equations and a few missed out the weight component, when using either method. In most cases answers were given to a suitable degree of accuracy.

5. There was a lot of confused work in part (a). Some candidates, not reading the question carefully enough, measured x from the surface of the earth and so could not successfully complete the argument, but many candidates had little idea of how to set up a relevant equation. Of those who gained all three marks it was probably evenly divided between those who started with $F = \frac{Gm_1m_2}{d^2}$ rather than the simpler $F = \frac{k}{x^2}$ suggested by the wording in the question.

In part (b) completely correct solutions were few and far between. Even for good candidates, who did appreciate that integration was needed, it was common to find either the minus sign missing from the equation of motion (producing $v = \sqrt{\frac{17gR}{6}}$) , or $x = 2R$ used instead of $x = 3R$ in the final stages (producing $v = \sqrt{\frac{gR}{2}}$). Weaker candidates often scored no marks here; it was common to see $v^2 = \frac{3}{2}gR + 2a(2R)$, with a constant value for a, or $\frac{1}{2}mv^2 = \frac{1}{2}m\frac{3gR}{2} + 2g(2R)$.

6. This proved a very good source of marks for the majority of candidates. The most common error was omission of the mass, but the mark scheme allowed 7 out of the 9 marks in this case. Most candidates did use $v\frac{dv}{dx}$ for acceleration, and apart from some errors in integration loss of marks was more usually from slips in manipulating the equations.

7. This question was very well-answered and there were many fully correct solutions. The most common errors were in the integration, omission of arbitrary constants or their incorrect evaluation. Some candidates used $v\ \frac{dv}{dt}$ for the acceleration and were heavily penalised.